Department of Higher Education
University of Computer Studies, Yangon
Fourth Year(B.C.Sc. / B.C.Tech.)
Final Examination
Mathematics of Computing IV (CST-402)
September, 2018

Answer ALL questions. **Time allowed : 3 hours.**

1(a)(i) In codeword Enumeration system, a computer considers a string of decimal digits a valid codeword if it contains an even number of '0' digits. For instance 1230407869 is valid, whereas 120987045608 is not valid. Let a_n be the number of valid n- digit codewords. Find a recurrence relation for a_n .

(ii) Find a recurrence relation for the number of ternary strings of length n that do not contain two consecutive 0s.What are the initial conditions and how many ternary strings of length nine do not contain two consecutive 0s?

(b) (i) Solve the recurrence relation $a_n = 7a_{n-1} - 10a_{n-2}$, $a_0 = 2$, $a_1 = 3$.

(ii) Find the closed form for the sequences, $\{a_k\}$ with $a_k = 3k$ and $a_k = 36k + 5$.

2(a) Find an explicit formula for the Fibonacci number $f_n = f_{n-1} + f_{n-2}$ with $f_0 = 0$ and $f_1 = 1$.

(b) Find the coefficient of x^{10} for the function $(1+x^5 + x^{10} + x^{15} + \ldots..)$

(c) Use generating functions to determine the number of different ways 25 identical apples can be given to 4 students if each student receives at least 3 but no more than 7 apples.

3(a) Use generating function to solve the recurrence relation $a_k = 3a_{k-1} + 4^{k-1}$, $a_0 = 1$.

(b) Let V = {S, A, B, a, b} and T = {a, b}. Find the language generated by the grammar (V, T, S, P) when the set P of productions consists of:

(i) S → AB, S → AA, A → aB, A → ab, B → b.

(ii) S → aS, S → bA, S → b, A → bA, A → b, and S → λ.

(c) Find a phrase-structure grammar for each of these languages.

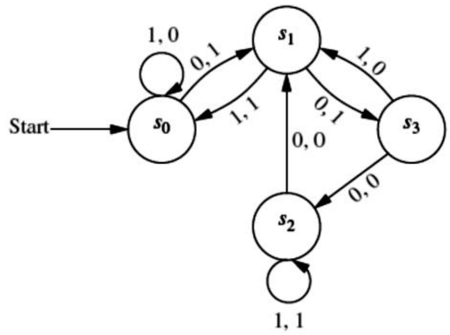
(i) $\{0^{2n}1^n | n = 0, 1, 2, 3, \ldots\}$

(ii) the set of bit strings that start with 11 and end with one or more 0s

4(a) Construct a finite-state machine that delays an input string one bit, giving **1** as the first bit of output, that is, it produces as output the bit string $\mathbf{1x_1x_2 \ldots x_{k-1}}$ given the input bit string $\mathbf{x_1x_2 \ldots x_k}$.

(b) Find the output for each of these input strings when given as input to the finite-state machine in given figure.

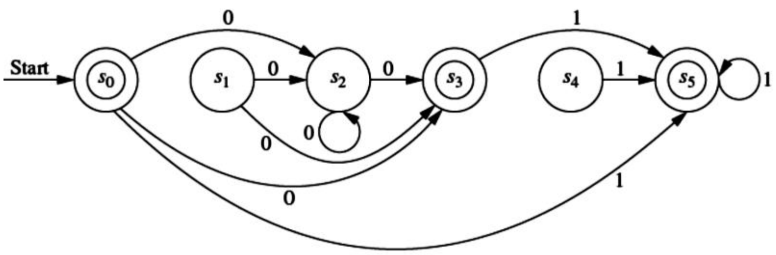
(i) 0111 (ii) 11011011 (iii) 01010101010 (iv) 10101010 (v) 01101111



(c) Construct a deterministic finite-state automaton that recognizes the set of bit string: $\{0^n, 0^n10x \mid n = 0, 1, 2, \ldots, \text{and x is any string}\}$.

(d) Construct a nondeterministic finite-state automaton that recognizes $\{0^n, 0^n01,\ 0^n011 | n \geq 0\}$

5(a) Find a regular grammar that generates the regular set recognized by the nondeterministic finite-state automaton shown in figure.



(b) Construct a Turing machine that computes the function f (n) = n − 2 if n ≥ 2 and f (n) = 0 for n = 0, 1 for all nonnegative integers n. For n=5, determine the final tape when it halts using this Turing machine. Show in details.

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